Proof

Before we proceed to find the values of a_n and b_n, we shall obtain the values of certain definite integrals, which are required in the evaluation of a_n and b_n.

$$\int_c^{c+2l} \cos\frac{n\pi x}{l}\,dx = \frac{l}{n\pi}\left(\sin\frac{n\pi x}{l}\right)_c^{c+2l}$$

$$= \frac{l}{n\pi}\left\{\sin\frac{n\pi}{l}(c+2l) - \sin\frac{n\pi c}{l}\right\}$$

$$= \frac{l}{n\pi}\left\{\sin\frac{n\pi c}{l} - \sin\frac{n\pi c}{l}\right\} = 0 \qquad (1)$$

$$\int_c^{c+2l} \sin\frac{n\pi x}{l}\,dx = -\frac{l}{n\pi}\left(\cos\frac{n\pi x}{l}\right)_c^{c+2l}$$

$$= -\frac{l}{n\pi}\left\{\cos\frac{n\pi}{l}(c+2l) - \cos\frac{n\pi c}{l}\right\}$$

$$= -\frac{l}{n\pi}\left\{\cos\frac{n\pi c}{l} - \cos\frac{n\pi c}{l}\right\} = 0 \qquad (2)$$

$$\int_c^{c+2l} \cos\frac{m\pi x}{l}\cos\frac{n\pi x}{l}\,dx = \frac{l}{2}\int_c^{c+2l}\left[\cos\frac{(m+n)\pi x}{l} + \cos\frac{(m-n)\pi x}{l}\right]dx$$

$$= 0,\ \text{if } m \neq n [\text{by}(1)] \qquad (3)$$

$$\int_c^{c+2l} \cos^2\frac{n\pi x}{l}\,dx = \frac{l}{2}\int_c^{c+2l}\left(1 + \cos\frac{2n\pi x}{l}\right)dx$$

$$= \frac{1}{2}\times 2l\ [\because \text{ the second term vanishes as in (1)}]$$

$$= l \qquad (4)$$

$$\int_c^{c+2l} \sin\frac{m\pi x}{l}\sin\frac{n\pi x}{l}\,dx = \frac{1}{2}\int_c^{c+2l}\left[\cos\frac{(m-n)\pi x}{l} - \cos\frac{(m+n)\pi x}{l}\right]dx$$

$$= 0,\ \text{if } m \neq n [\text{by}(1)] \qquad (5)$$

$$\int_c^{c+2l} \sin^2\frac{n\pi x}{l}\,dx = \frac{1}{2}\int_c^{c+2l}\left(1 - \cos\frac{2n\pi x}{l}\right)dx$$

$$= \frac{1}{2}\times 2l\ [\because \text{ the second term vanishes as in (1)}]$$

$$= l \qquad (6)$$